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Harbor Protection Project
Yale University
New Haven, Connecticut

Technical Memorandum No. 5 (Revised)
(HPP:411:Ser 003)
18 April 1952

Vortex Sweepings

by C. T. Lane

SECRET

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COPY NO. 2

54AA-10259

VORTEX SWEEPINGS

Reference a
E. Hill and R. Richtmeyer, BuShips Tech. Rep. No. 51 (1942)
Reference b
H. Lamb, "Hydrodynamics", 4th Ed. Camb. Univ. Press (1916)
Reference c
L. Page, HPP:400LP:Ser 003 (23 June 1951)
Reference d
AeroJet Proposal - verbal report by L. W. McKeehan (1951)
Reference e
Osborne Reynolds, Nature, vol. 14, p. 477 (1876)

Preliminary Remarks

The suggestion that the pressure deficiency, created by a vortex in water, might be used to sweep pressure mines apparently originated in 1942 (Reference a). This very good report, entirely theoretical, has not, apparently, received as much attention as it deserves. This may be due, in part, to the fact the report is very condensed, lacks sufficient reference to source material and omits a number of crucial mathematical proofs.

I have, accordingly, re-studied the whole situation against the basic theory (Reference b) and rectified, in the Appendix to this document, some of the omissions. It is hoped that these two reports, taken together, will form a reasonably satisfactory background of the problem. In what follows the two methods so far suggested for practical development are discussed and in addition some further suggestions made.

(1) Page System

This suggestion (Reference c) was made independently of a

similar one in Reference a. In principle, we are to generate a cylindrical vortex (i.e. cyclone type) extending from the surface to the bottom of the water. (Figure 1) That a pressure deficiency, in comparison with the hydrostatic pressure on the bottom, must exist follows from elementary considerations. Thus, within the vortex, each cubic centimeter of the whirling fluid experiences a force (centripetal force) in a direction <u>towards</u> the axis of rotation. Accordingly there must be a <u>decrease</u> in pressure in going toward the axis.

Calculation (Reference c) on the energy required to produce such a vortex with a diameter of around 200 feet and a pressure deficiency at the axis of 3 in. water indicate about 32 horsepower-hour for an ideal * (i.e. zero viscosity) liquid. Even if this figure were multiplied by a factor of ten to allow for the viscosity of water it would still be quite reasonable.

However, in experiments to date, we have been able to produce such vortex only on a small scale. All attempts to produce them for instance in a swimming pool (depth $\sim$ 10 feet) have, so far, failed. The theory is of little help in this respect - it gives us practically no hints as to how a vortex can be initiated, but merely assumes they exist and then predicts their behaviour. Assuming that a vortex can eventually be produced, there is another factor which would be of some importance. Namely, it would clearly be desirable to either tow or else cast the vortex free from the generating apparatus. Suggestions

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* The hydrodynamic theory applies only to such liquids.

on this point emerge from the theory and are shown in Figure 2.

In summary then, the crux of the situation is whether such a large scale vortex can actually be produced in practice - if it can the method would appear to be an excellent one for mine sweeping, but so far the experiments have not been promising.

(2) Aerojet System

This method, recently proposed by the Aerojet Co. (Reference d), had also been proposed earlier (Reference a). In essentials, it is as follows:

A vortex ring (i.e. "smoke-ring" type) is projected vertically down below the surface, such that the plane of the ring is parallel to the bottom. Upon approaching the bottom the ring will expand in diameter, with its plane still parallel, due to its reaction with the solid surface. In so doing it will cause a flow of water along the bottom which, by the Bernoulli effect, will cause a pressure deficiency there, which will move with the expanding ring.

At first sight this proposal appears most attractive. For one thing such ring vortices (at least on a small scale) are readily produced in water (Reference e). As early as 1876 Osborne Reynolds was able to "fire" such rings (diameter $\sim$ 1 inch) in water * for distances up to twenty feet without very much change in size. Also, the theory of such vortices (for the ideal liquid) has been fairly completely worked out (Reference b).

* By using colored water in the rings they were rendered visible in uncolored water.

However, when we calculate (Reference a) the minimum energy and momentum required to sweep a mine * serious objections arise. It appears (see Appendix) that the momentum carried by such a ring (diameter $\sim$ 6 ft.), and hence the recoil momentum which would have to be absorbed by the launching apparatus, would not be less than that due to the blast of a sixteen inch gun. In addition, it seems very probable that the peak power (which is, of course, quite different from the average power) required to produce one ring would be in the neighborhood of 500,000 horsepower.

Naturally this does not prove the method impossible; it merely points up the fact that its practical realization will be beset with some formidable difficulties. Of course much depends on how far we can rely on the theoretical estimates. No exact solution of the problem is possible - we are forced, by its complexity, to use approximate methods. I believe the above figures are conservative and have been at considerable pains to check the formulae used in Reference a against the basic theory developed in Reference b. The very large figures given arise, essentially, for two separate reasons:

(i) In order to produce the required ΔP for the necessary Δt very large vortex strengths are required. In other words, the fluid must be given very high rotational speeds. This leads to a large

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* Assumed that it is necessary to produce a pressure deficiency (ΔP) of 4 ins. water for a period of time (Δt) of 12 sec.

kinetic energy for the vortex.

(ii) This kinetic energy, although large, is not impossibly so. However, if, as appears probable, the time of formation of the ring must also be short ($\sim \frac{1}{50}$ sec) this leads to the very high peak power and recoil force noted above.

In other words, if some way could be found to produce such ring vortices slowly (i.e. in times of the order of seconds) the above objections would largely disappear. The time of formation of the ring is therefore pretty nearly the crux of the problem. As far as I can see, no very reliable calculation of the time of formation of the ring can be made from the existing theory - in Reference a it is estimated as $\sim \frac{1}{100}$ sec. This is a matter of experimentation, but such experiments must be carefully executed and interpretated.

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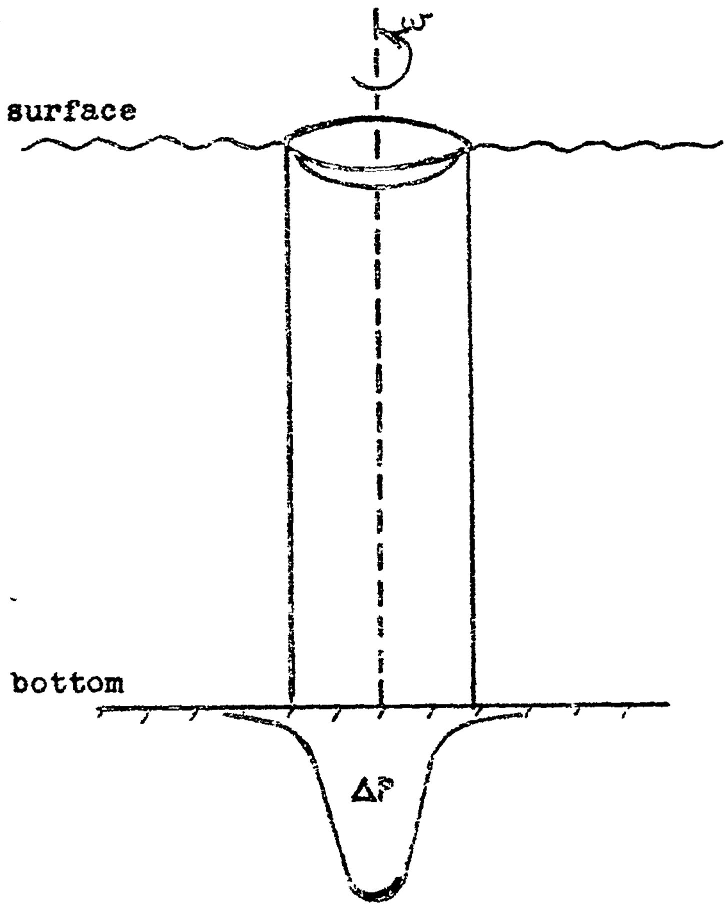
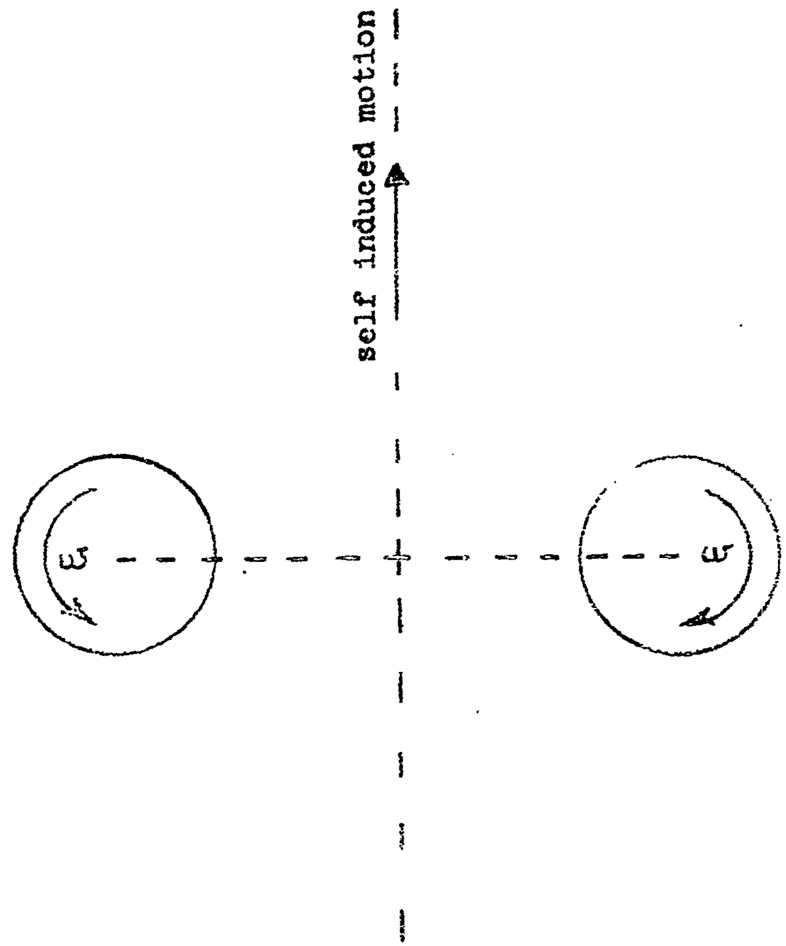
FIGURE 1



Cylindrical vortex with pressure deficiency (ΔP) inside



Two vortices of equal strength rotating in opposite sense will be self-propelled

FIGURE 2

FIGURE 3

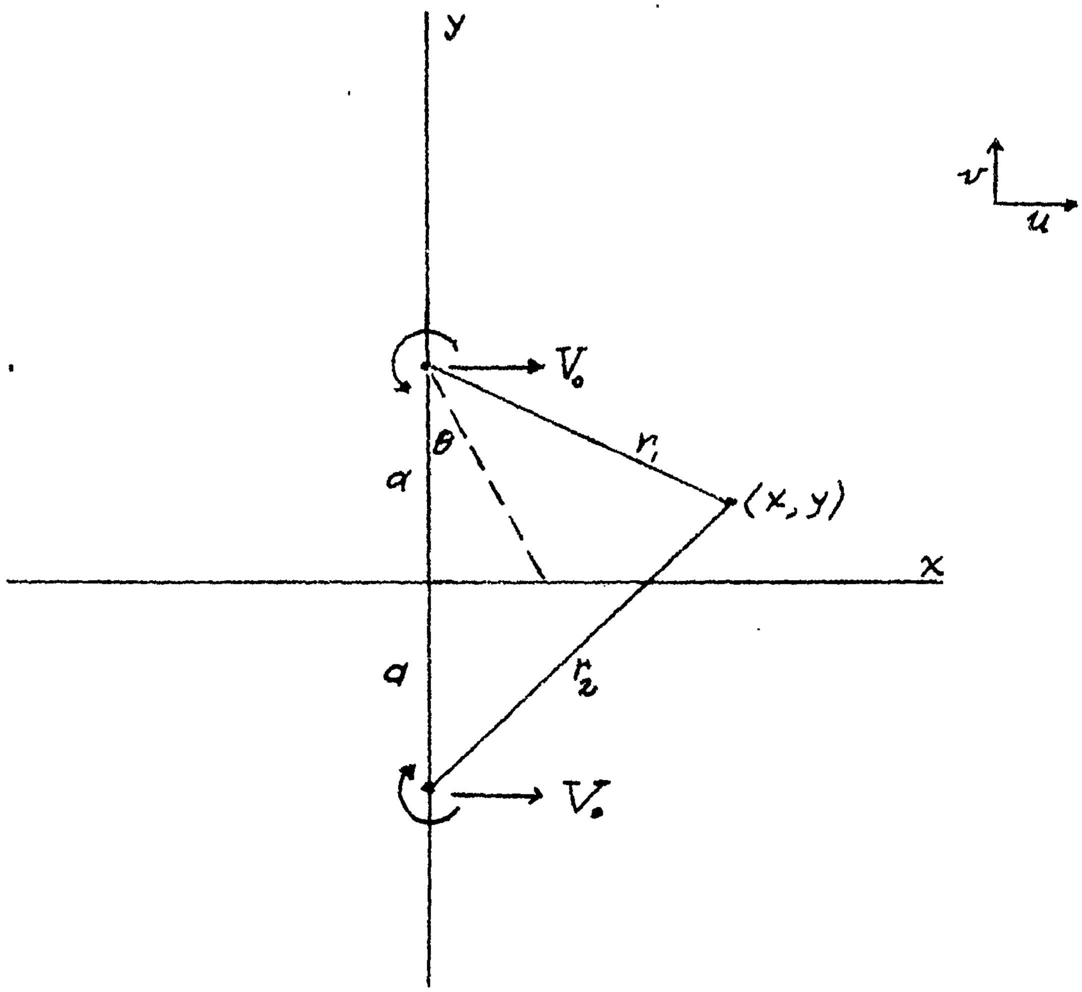
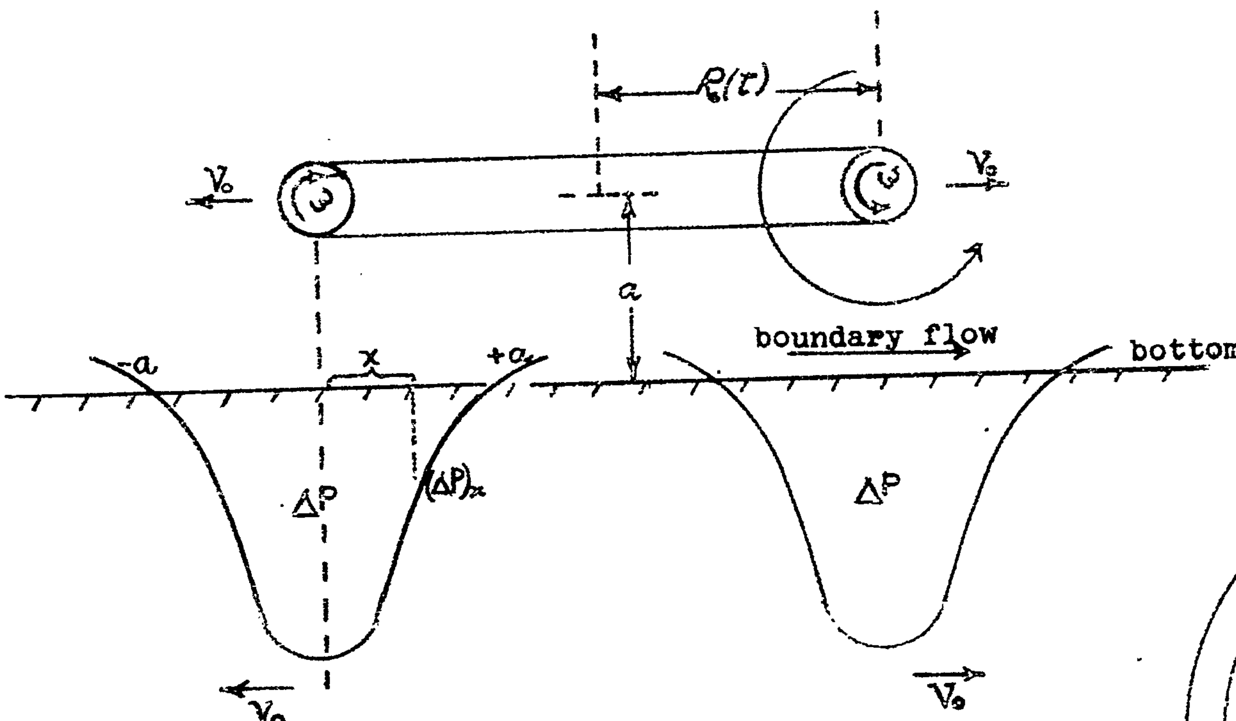
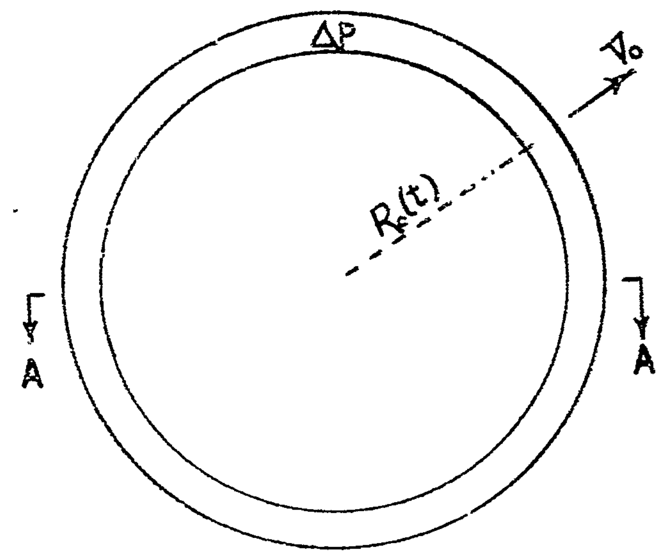


FIGURE 4



Sect A-A



Ring vortex near a wall and expanding laterally with vel. V_o showing the expanding "pressure deficiency trough" (ΔP) induced by the boundary flow via the Bernoulli effect

Appendix

Symbols

R_o = mean radius of the ring

r_o = radius of the toroidal cross section

ω = angular velocity of fluid (constant over r_o)

$\mathcal{E}$ = kinetic energy of vortex

M = momentum of vortex

κ = vortex strength ($\pi r_o^2 \omega$)

ρ = density of the liquid

ψ = stream function

Δp = pressure deficiency

Δt = time of duration

V = linear velocity of ring along axis (perp. to its plane) through center

r_1, r_2 = position vectors

x, y = point co-ordinates in a plane

a = distance of ring from the bottom

The following pertinent relations for a ring vortex for which $R_0 \gg r_0$ are derived in Reference b (p. 233) and copied in Reference a:

$$1. \quad \mathcal{E} = \frac{\rho \kappa^2 R_0}{2} \left\{ \ln \frac{8R_0}{r_0} - \frac{7}{4} \right\}$$

$$2. \quad V = \frac{\kappa}{4\pi R_0} \left\{ \ln \frac{8R_0}{r_0} - \frac{1}{4} \right\}$$

The momentum of the ring is obtained from Reference b, p. 231 equation (3) as

$$3. \quad M \cong \pi \rho \kappa R_0^2$$

A cross section through the ring showing the stream lines is pictured in Reference b page 230. The case of a ring approaching a wall which is parallel to its plane (i.e. the bottom) is discussed in Reference b and, more completely, in Reference a. As the ring approaches the wall it slows down and spreads out laterally (i.e. R_0 increases with time) with a velocity (V_0) parallel to the wall:

$$4. \quad V_0 = \frac{\kappa}{4\pi a}$$

This produces a (radial) flow of fluid along the wall and consequently, by Bernoulli's theorem, a pressure deficiency (Δp) there. The rigorous calculation of the velocity (u) of the fluid along the wall is an impossibly difficult mathematical problem and recourse must be had to a somewhat rough approximation. Namely, the problem is reduced to one in two dimensions by considering a linear vortex filament and

its image moving parallel to the wall with velocity V_0 at a distance a (Fig.3). As shown in Reference b, the motion of such an element may be considered in terms of an image element of equal and opposite vorticity at distance "a" behind the wall. The element and its image then move parallel to the wall with a velocity $\kappa/4\pi a$ at a constant distance $2a$ apart.

The stream function (Reference b, page 216) is then:

$$5. \qquad \psi = \frac{\kappa}{2\pi}\left\{\frac{y}{2a} + \ln\frac{r_1}{r_2}\right\}$$

This is a relative stream function i.e. it takes account of the linear motion of the vortex elements, by impressing on the reference frame a velocity V_0 in the positive x- direction. Clearly the x-axis represents a stream line ($\psi = 0$). In order to compute the velocity of the fluid near the x-axis (i.e. the wall) we proceed as follows:

Since

$$6. \qquad u = -\frac{\partial\psi}{\partial y} \qquad v = \frac{\partial\psi}{\partial x}$$

where u, v are the velocities in the x and y direction respectively we consider a point (x, y) very near the wall. From Fig. 3 then we have

$$r_1^2 = x^2 + (a-y)^2$$
$$r_2^2 = x^2 + (a+y)^2$$

i.e.

$$\ln\frac{r_1}{r_2} = \frac{1}{2}\ln\frac{x^2+(a-y)^2}{x^2+(a+y)^2}$$

Whence

$$u = -\frac{\partial\psi}{\partial y} = -\frac{K}{2\pi}\left[\frac{1}{2a} - \frac{\left\{x^2+(a+y)^2\right\}(a-y)+\left\{x^2+(a-y)^2\right\}(a+y)}{\left\{x^2+(a-y)^2\right\}\left\{x^2+(a+y)^2\right\}}\right]$$

If now we let $y \to 0$ we get

$$7.\quad u = -\frac{K}{2\pi}\left[\frac{1}{2a} - \frac{2a}{a^2+x^2}\right] = \frac{K}{4\pi a}\left[\frac{4a^2}{a^2+x^2} - 1\right] = \frac{K}{4\pi a}\left(4\cos^2\theta - 1\right).$$

A similar calculation shows that $v = 0$, hence the velocity of the fluid is entirely tangential at the wall. We may now proceed to compute the pressure deficiency (Δp) comparing the region near the vortex element with regions remote from it where the pressure is p_0. Now when $\theta = \frac{\pi}{2}$ i.e., when $x = \infty$, clearly $u = -V_0$ from the above equation. This means that the reference frame to which u is referred has impressed on it a velocity V_0 in the positive x-direction -- a fact, already mentioned, implicit in the stream function ψ from which u is derived. In applying Bernoulli's theorem we must do the same for those regions remote from the vortex (p_0).

Hence

$$8.\quad \frac{p}{\rho} + \frac{u^2}{2} = \frac{p_0}{\rho} + \frac{V_0^2}{2}$$

i.e. $$9.\quad \frac{\Delta p}{\rho} = \frac{u^2}{2} - \frac{V_0^2}{2} = \frac{V_0^2}{2}\left\{\left(4\cos^2\theta - 1\right)^2 - 1\right\} = 4V_0^2\cos^2\theta\left(2\cos^2\theta - 1\right)$$

Clearly this function has a maximum at $\theta = 0$ ($x = 0$) and is zero at $\theta = 45°$ ($x = \pm a$) and at $x = \pm\infty$. At $x = \pm\frac{a}{2}$, $\Delta p = 2\rho V_0^2$ very nearly -- hence we may choose this value for sweeping, noting that it is at least this amount over the interval $-\frac{a}{2} \leq x \leq \frac{a}{2}$. Calling this value $\overline{\Delta p}$, if the necessary time of application be $\overline{\Delta t}$ then

$$\overline{\Delta t} = \frac{a}{V_0} = \frac{a}{\frac{\overline{\Delta p}}{2\rho V_0}} = \frac{2a\rho V_0}{\overline{\Delta p}} = \frac{K\rho}{2\pi\overline{\Delta p}}$$

i.e. 10. $\overline{\Delta p} \cdot \overline{\Delta t} = \frac{K\rho}{2\pi}$

Equation (10) permits us to estimate the strength of vortex required to sweep a mine, for $\overline{\Delta p} = 4$ inches water, $\overline{\Delta t} = 12$ sec.

$$K \sim 10^6 \text{ cm}^2/\text{sec}$$

Using equation (3) the momentum for a ring $R_0 = 100$ cm. (diameter $\sim$ 6 ft.) is

$$M \sim \pi \cdot 1 \cdot 10^6 \cdot (100)^2 \sim 10^{11} \text{ dyne} \cdot \text{sec.}$$

This is of the order of the recoil momentum, on discharge, of a 16-inch gun.*

Again, from equation (1) the kinetic energy of the ring will be:

$$\mathcal{E} \sim \rho K^2 R_0 \sim 10^{14} \text{ ergs} \qquad (R_0 \sim 10 r_0)$$

In the event that the ring must be produced in a time $\sim \frac{1}{50}$ sec. this will require a peak power of:

$$\sim 5 \times 10^{15} \text{ ergs/sec} \sim 500{,}000 \text{ HP}$$

Finally the velocity of the ring toward the bottom will be:

$$V \sim \frac{K}{\pi R_0} \sim 3 \times 10^3 \text{ cm/sec} \sim 100 \text{ ft/sec}$$

For larger ring diameters the momentum and energy will be increased and the launching velocity decreased.

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* Assumed weight of shell 1 ton, muzzle velocity 3000 ft/sec. Also assumed that the time of discharge of the shell is the same order as the time of formation of the vortex ring ($\sim \frac{1}{50}$ sec.).